$$y = F(x)$$

$$y^2 = [F(x)]^2$$

$$2y^1 \cdot \frac{dy}{dx} = 2[F(x)]^1 F'(x)$$

Explanation of how the chain rule works in implicit differentiation.

8) $\sqrt{xy} = x - 2y$

$$x^{\frac{1}{2}} y^{\frac{1}{2}} = x - 2y$$

$$\frac{1}{2}x^{-\frac{1}{2}} y^{\frac{1}{2}} + x^{\frac{1}{2}} \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx} = 1 - 2\frac{dy}{dx}$$

$$\frac{\sqrt{y}}{2\sqrt{x}} + \frac{\sqrt{x}}{2\sqrt{y}} \frac{dy}{dx} = 1 - 2\frac{dy}{dx}$$

$$\left(\frac{\sqrt{x}}{2\sqrt{y}} + 2\frac{2\sqrt{y}}{2\sqrt{y}}\right)\frac{dy}{dx} = \frac{2\sqrt{x}}{2\sqrt{x}}1 - \frac{\sqrt{y}}{2\sqrt{x}}$$

$$\frac{\sqrt{x} + 4\sqrt{y}}{2\sqrt{y}} \frac{dy}{dx} = \frac{2\sqrt{x} - \sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{2\sqrt{x} - \sqrt{y}}{\cancel{2}\sqrt{x}} \cdot \frac{\cancel{2}\sqrt{y}}{\sqrt{x} + 4\sqrt{y}}$$

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10) $2\sin x \cos y = 1$

$$2\cos x \cos y + 2\sin x\left[-\sin y \frac{dy}{dx}\right] = 0$$

$$\cancel{2}\cos x \cos y = \cancel{2}\sin x \sin y \frac{dy}{dx}$$

$$\frac{\cos x \cos y}{\sin x \sin y} = \frac{dy}{dx}$$

$$\cot x \cot y = \frac{dy}{dx}$$

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12) $(\sin \pi x + \cos \pi y)^2 = 2$

$$\cancel{2}(\sin\pi x + \cos\pi y)^1(\pi\cos\pi x - \pi\sin\pi y\frac{dy}{dx}) = 0$$

$$\pi\sin\pi x\cos\pi x - \pi\sin\pi x\sin\pi y\frac{dy}{dx} + \pi\cos\pi x\cos\pi y - \pi\cos\pi y\sin\pi y\frac{dy}{dx} = 0$$

$$\pi\sin\pi x\cos\pi x + \pi\cos\pi x\cos\pi y = (\pi\sin\pi x\sin\pi y + \pi\cos\pi y\sin\pi y)\frac{dy}{dx}$$

$$\frac{\pi\cos\pi x(\sin\pi x + \cos\pi y)}{\pi\sin\pi y(\sin\pi x + \cos\pi y)} = \frac{dy}{dx}$$

$$\frac{\cos\pi x}{\sin\pi y} = \frac{dy}{dx}$$

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14) $\cot y = x - y$

$$-\csc^2 y\frac{dy}{dx} = 1 - 1\frac{dy}{dx}$$

$$(1-\csc^2 y)\frac{dy}{dx} = 1$$

$$-\cot^2 y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = -\tan^2 y$$

16) $x = \sec\frac{1}{y}$ $\quad y^{-1}$

$$1 = \left[\sec\frac{1}{y}\tan\frac{1}{y}\right]\left(-y^{-2}\frac{dy}{dx}\right)$$

$\Updownarrow$

$$\frac{0(y) - 1(1\frac{dy}{dx})}{y^2}$$

$$1 = \sec\frac{1}{y}\tan\frac{1}{y}\left(-\frac{1}{y^2}\right)\frac{dy}{dx}$$

$$\frac{-y^2}{\sec\frac{1}{y}\tan\frac{1}{y}} = \frac{dy}{dx}$$

$$-y^2\cos\frac{1}{y}\cot\frac{1}{y} = \frac{dy}{dx}$$

Quotient Rule
vs
Power Rule

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22) $x^2 - y^3 = 0 \qquad (1,1)$

$$2x - 3y^2\frac{dy}{dx} = 0$$

$$\frac{2x}{3y^2} = \frac{dy}{dx}\Big|_{(1,1)} = \frac{2(1)}{3(1)^2} = \frac{2}{3}$$

24) $(x+y)^3 = x^3 + y^3 \qquad (-1,1)$

$$3(x+y)^2\left(1+\frac{dy}{dx}\right) = 3x^2 + 3y^2\frac{dy}{dx}$$

$$\cancel{3(-1+1)^2\left(1+\frac{dy}{dx}\right)} = 3(-1)^2 + 3(1)^2\frac{dy}{dx}$$

$$-3 = 3\frac{dy}{dx} = -1$$

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26) $x^3 + y^3 = 4xy + 1 \qquad (2,1)$

$$3x^2 + 3y^2\frac{dy}{dx} = 4y + 4x\frac{dy}{dx} + 0$$

$$(3y^2 - 4x)\frac{dy}{dx} = 4y - 3x^2$$

$$\frac{dy}{dx} = \frac{4y-3x^2}{3y^2-4x}\Big|_{(2,1)} = \frac{-8}{-5} = \frac{8}{5}$$

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